Because of the three-axis stability requirement, I_2 must be greater than I_1, which is equivalent to $\lambda_1^0 > \lambda_2^0$. Consequently, the resonant lengths for the roll motion are shorter than the resonant lengths for the pitch motion. The results, shown in Fig. 2, are summarized in the following inequalities:

$$L < \frac{\pi}{2}\sqrt{\frac{I_1}{\rho_m A\rho}}, \qquad \frac{\pi}{2}\sqrt{\frac{I_2}{\rho_m A\rho}} < L < \pi\sqrt{\frac{I_1}{\rho_m A\rho}}$$

$$\pi\sqrt{\frac{I_2}{\rho_m A\rho}} < L < \frac{1}{\Omega}\sqrt{\frac{EAI_1}{12m^2\rho}} \tag{22}$$

Referring to Fig. 2, tether lengths $L < 250m$ and $L > 1km$ are safe because they are clear of unmodeled frequencies. Another interesting point is that the pitch and the roll stiffness are equal when $4K_1 + 3\lambda_1 = 3K_2 + 3\lambda_2$. In case of large stiffness control parameters, this equality implies $\lambda_1 = \lambda_2$ or $I_1 = I_2$. This condition must be avoided to prevent a possible nonlinear resonance in pitch–roll. The general condition $k(4K_1 + 3\lambda_1) = l(3K_2 + 3\lambda_2)$, for integers k and l, may also lead to natural resonances. A tethered gravity-gradient satellite is free of this problem because the frequencies $\sqrt{3}$ and 2 are well separated and are noncommensurate.

III. Concluding Remarks

The passive attitude stabilization by means of a tether appears promising for Earth-pointing satellites with moderate-to-high pointing accuracies. The preliminary investigation presented in this Note points out the following main results.

The control/restoring torque is strong enough for the pitch–roll responses. A linear controller cannot provide a direct yaw control torque. The yaw control may be designed by using the linear or the nonlinear coupling or by installing a reaction wheel if a better yaw response is required. The pitch is very stable, even for configurations with $I_3 > I_2 > I_1$, as opposed to a nontethered gravity-gradient stabilized satellite. The three-axis stability region covers most of the plane $I_2 > I_1$. The DeBra–Delp gyric stability disappears. Moreover, there is a tradeoff to be made between stability and disturbance rejection when the tether length is selected. Although a long tether provides large restoring torque, a short tether is better for filtering out high frequencies. A long tether makes the system very stiff and sensitive to external disturbances. The pointing accuracy may be better for a longer tether in an unperturbed circular orbit, but unavoidable environmental perturbations acting on the tether will be transmitted to the satellite and affect its pointing accuracy. Typical satellite-tether configurations are outside the orbital eccentricity resonance region. However, parametric resonances as a result of interaction between the satellite and the tether flexible modes may occur for some configurations and must be examined carefully for each design.

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Aircraft Trajectory Optimization in the Horizontal Plane

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I. Introduction

OPTIMIZATION of atmospheric flight trajectories has been of great interest for many decades. Time-optimal trajectories in the horizontal plane have been investigated either with soft control constraints,[1,2] via a quadratic penalty in the cost, or with hard constraints,[3–5] i.e., with strictly bounded control functions. The prevailing optimization method has been the minimum principle.

This work considers the planar time-optimal trajectory problem with hard control constraints and under the assumption of constant aircraft velocity, which simplifies the problem considerably.[4] The case with no wind is investigated thoroughly in Ref. 3, where it is shown that the optimal trajectories are composed of bang, i.e., maximum turn rate, and singular, i.e., level flights with zero turn rate, segments with a maximum number of four turns. Shapira and Ben-Asher[5] proposed the problem of introducing winds into the formulation. Unfortunately, closed-form expressions have not been obtained in the presence of winds, even for the restricted bang–singular–bang trajectories[5] (a simple case recommended in Ref. 3 for practical applications).

The main objective of the present research is to develop an optimization technique to obtain planar time-optimal trajectories in the presence of winds. As opposed to all previous publications on this topic, the approach taken is not based on the minimum principle. Instead, it employs a parameterization technique, originally developed for a class of bilinear systems,[6] which fits nicely to the problem of interest. The technique transforms the problem into a finite dimensional optimization problem.

The problem is formulated in Sec. II. Section III analyzes the problem and makes the mentioned transformation to the finite dimensional space. Finally, representative numerical results that manifest the nature of the solution are given in Sec. IV, and the conclusions are drawn in Sec. V.

II. Problem Formulation

Assuming constant velocity (more precisely, constant true air speed) and constant altitude, the aircraft equations of motion in the horizontal plane can be written as

$$\dot{x}(t) = V\cos[\sigma(t)] + W_x(t)$$
$$\dot{y}(t) = V\sin[\sigma(t)] + W_y(t), \qquad \dot{\sigma}(t) = u(t) \tag{1}$$

where $x(t)$ and $y(t)$ are displacements with respect to the ground, V is the true air speed, and $W_x(t)$ and $W_y(t)$ are components of the wind velocity. No position dependence, i.e., homogeneous wind field, is assumed; $\sigma(t)$ is the heading angle of the aircraft; and $u(t)$ is the control variable. We assume that $u(t)$ is constrained by $u(t) \in [-b, b]$.

The optimization problem is to find the control time history, which drives the system (1) from a given initial condition $\{x(0) = x_0, y(0) = y_0, \sigma(0) = 0\}$ to a required target point $\{x(T) = x_T, y(T) = y_T, \sigma(T) = \sigma_T \geq 0\}$ while minimizing the transition time T.

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Notice that the final heading also is given, as is the case in aircraft approaching a landing runway and other applications. For simplicity, we restrict the discussion to nonnegative σ_T. For negative final-heading cases, the analysis is virtually the same, with some minor changes.

III. Problem Analysis

Assuming constant wind velocities, by integrating Eq. (1), we get

$$x(T) = x_0 + V \int_0^T \cos[\sigma(t)]\,\mathrm{d}t + W_x T$$
$$y(T) = y_0 + V \int_0^T \sin[\sigma(t)]\,\mathrm{d}t + W_y T \tag{2}$$

Remark: We can consider any integrable functions $[W_x(\cdot), W_y(\cdot)]$ defined on the time segment instead of the constant wind components and continue along the same lines. Notice, however, that the important case of position-dependent winds is not considered here. Let us introduce the following replacement:

$$\sigma(t) = \int_0^t u(\tau)\,\mathrm{d}\tau = \int_0^T u(\tau)\,\mathrm{d}\tau - \int_t^T u(\tau)\,\mathrm{d}\tau = \sigma_T - z(t) \tag{3}$$

We have to find $u(\cdot)$ such that

$$\int_0^T \Phi[z(t)]\,\mathrm{d}t = \Omega, \qquad \Phi(z) = \begin{bmatrix} V\cos(\sigma_T - z) \\ V\sin(\sigma_T - z) \end{bmatrix}$$
$$\Omega = \begin{bmatrix} x_T - x_0 - W_x T \\ y_T - y_0 - W_y T \end{bmatrix} \tag{4}$$

for the possible minimum T.

Notice the properties of the absolutely continuous function $z(\cdot) : [0, T] \mapsto R$, defined by

$$z(t) = \int_t^T u(\tau)\,\mathrm{d}\tau$$

One can see that $z(0) = \sigma_T$, $z(T) = 0$, and $\dot{z}(t) \in [-b, b]$ a.e. Let Z_T be the set of all scalar absolutely continuous functions $z(\cdot)$ defined on $[0, T]$ such that $z(0) = \sigma_T$, $z(T) = 0$, and $\dot{z}(t) \in [-b, b]$ a.e. Let Zl_T be the set of all piecewise linear functions $z(\cdot) : [0, T] \mapsto R$ such that $z(0) = \sigma_T$, $z(T) = 0$, and $\dot{z}(t) \in \{-b, 0, b\}$ a.e. One can show that Zl_T is dense in Z_T, i.e., any function in Z_T can be approximated with any desired precision by a function in Zl_T. Thus, we can consider our problem on the set Zl_T (instead of Z_T). Moreover, we have the following theorem.[6,7]

Theorem: Let $\Phi(\cdot)$ be a continuous n-dimensional function, i.e., $\Phi(\cdot) : R \mapsto R^n$; then, for any $z(\cdot) \in Z_T$, there exists $\bar{z}(\cdot) \in Zl_T$ such that

$$\int_0^T \Phi[z(t)]\,\mathrm{d}t = \int_0^T \Phi[\bar{z}(t)]\,\mathrm{d}t \tag{5}$$

and the number of switches {a switch is defined by a point in time where $[\mathrm{d}\bar{z}(t)]/\mathrm{d}t$ changes its value} is no more than $2(n + 2)$.

Proof: The proof is given in Refs. 6 and 7.

For our case, $n = 2$; therefore, the number of switches is no more than $2(n + 2) = 8$ and the number of zero-slope segments is no more than three. Moreover, it can be proved[7] that Fig. 1 describes the general solution to this problem, where z_m, z_M are the minimum and maximum values of $\bar{z}$, respectively, and any other structure can be transformed to an equivalent one of this form, with identical terminal time. Let τ_i, z_i denote the duration and the value, respectively, of the ith zero-slope segment. It is easy to see that

$$\Omega = \frac{1}{b}\left[\int_{z_m}^0 \Phi(\bar{z})\,\mathrm{d}\bar{z} + \int_{\sigma_T}^{z_M} \Phi(\bar{z})\,\mathrm{d}\bar{z} + \int_{z_m}^{z_M} \Phi(\bar{z})\,\mathrm{d}\bar{z}\right] + \sum_{i=1}^{3} \tau_i \Phi(z_i) \tag{6}$$

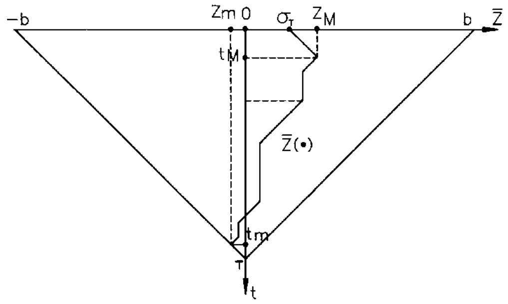

Fig. 1 Piecewise linear optimal solution.

where $\Phi(\cdot)$ is defined in Eq. (4) and where the variables $\{\tau_i, z_i, z_m, z_M, T\}$ should satisfy

$$z_M \in \left[\sigma_T, \frac{\sigma_T + bT}{2}\right], \qquad z_m \in \left[\frac{\sigma_T - bT}{2}, 0\right]$$
$$z_M - z_m \in \left[\sigma_T, \frac{\sigma_T + bT}{2}\right], \qquad z_i \in [z_m, z_M]$$
$$\tau_i \geq 0, \qquad \sum_{i=1}^{3} \tau_i = \frac{(\sigma_T + bT) - 2(z_M - z_m)}{b}$$

Substituting Eq. (4) into Eq. (6) and performing the integration, we obtain

$$\Omega = \frac{V}{b}\begin{bmatrix} 2\sin(\sigma_T - z_m) - 2\sin(\sigma_T - z_M) - \sin(\sigma_T) \\ -2\cos(\sigma_T - z_m) + 2\cos(\sigma_T - z_M) + \cos(\sigma_T) - 1 \end{bmatrix} + \sum_{1}^{3} V\tau_i \begin{bmatrix} \cos(\sigma_T - z_i) \\ \sin(\sigma_T - z_i) \end{bmatrix} \tag{7}$$

The problem is, therefore, to find $\{\tau_i, z_i, z_m, z_M, T\}$ (may not be unique) that satisfy Eq. (7) while minimizing T. Notice that this is a parameter optimization problem with nine unknowns. There are 15 linear inequality constraints, 9 of which are simple bounds on the parameters, and 3 simple linear equality constraints. Finally, there are two more nonlinear equality constraints [Eq. (7)] that render the programming problem nonlinear.

Remark: In comparison to the results of Ref. 3, obtained by more standard methods, our method increases the number of unknown variables from four (as required by Ref. 3) to nine. However, the proposed technique solves the problem with virtually the same effort, even when winds are present, a case not considered in Ref. 3. Moreover, the nine variables parameterize all possible trajectories, whereas the four variables are sufficient for specific patterns, e.g., bang–bang–bang, bang–singular–bang.

Having solved this problem, one can easily construct the corresponding function $\bar{z}(\cdot)$ as in Fig. 1 (with z_m its minimum, z_M its maximum, and z_i the constant values along time intervals of length τ_i) such that the function $\bar{\sigma}(t) = \sigma_T - \bar{z}(t)\; t \in [0, T]$ is piecewise linear:

$$\frac{\mathrm{d}\bar{\sigma}(t)}{\mathrm{d}t} \in \{-b, 0, b\}$$

has no more than eight switching moments, and the trajectory generated by the control

$$\bar{u}(t) = \frac{\mathrm{d}\bar{\sigma}(t)}{\mathrm{d}t}$$

satisfies Eq. (2).

IV. Numerical Examples

We search for optimal trajectories for a vehicle characterized by $V = 984$ ft/s, $b = 0.2$ rad/s, with initial position at the origin and a required target point $\{x_T = 8000$ ft, $y_T = 14{,}500$ ft, $\sigma_T = 0\}$. Notice that multiples of 2π also are considered as legitimate terminal headings.

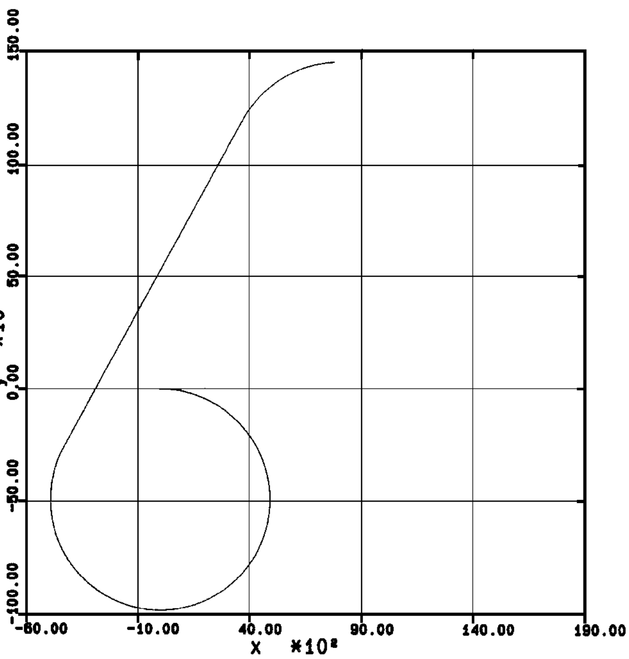

Fig. 2 Bang–level–bang trajectory, without wind (suboptimal).

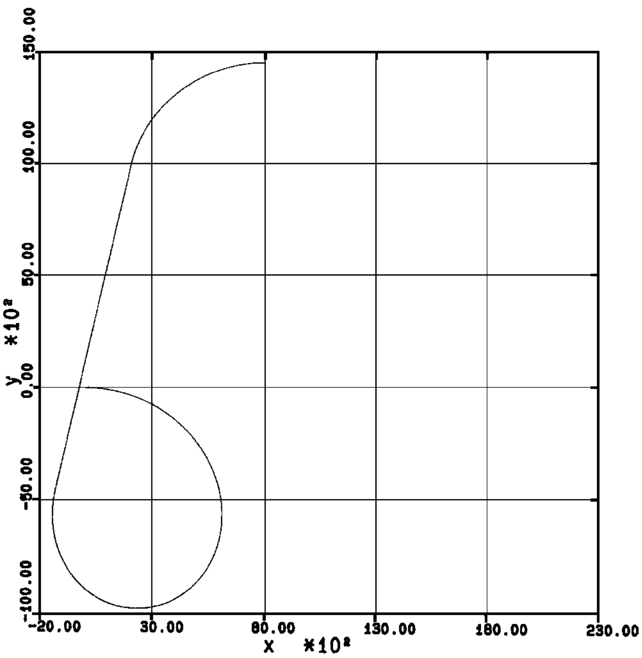

Fig. 4 Bang–level–bang trajectory, with wind (optimal).

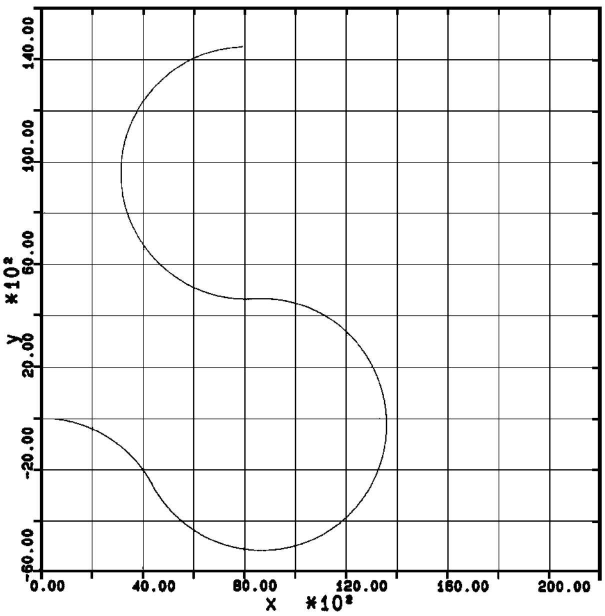

Fig. 3 Bang–bang–bang trajectory, without wind (optimal).

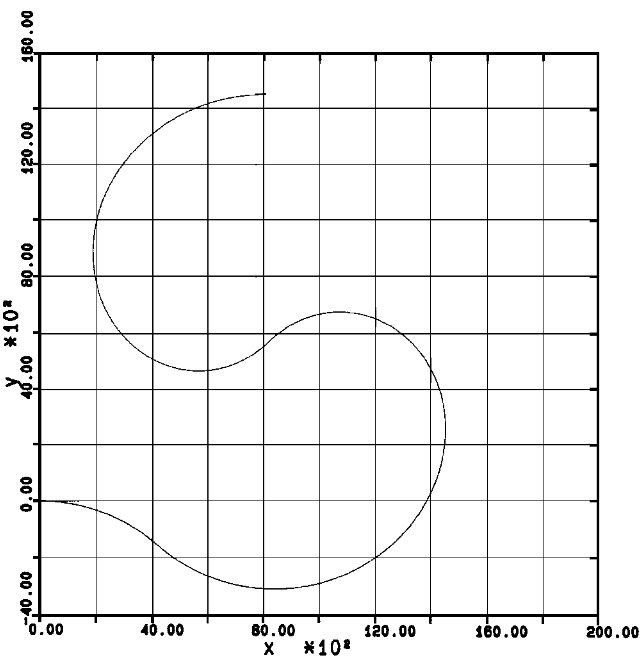

Fig. 5 Bang–bang–bang trajectory, with wind (suboptimal).

The MATLAB® Optimization Toolbox is employed to find the optimal set of parameters $\{\tau_i, z_i, z_m, z_M, T\}$. The numerical code is based on the sequential quadratic programming method, and it may only give local solutions. The convergence was extremely fast (only a few CPU seconds on a Pentium processor compared to a few CPU minutes required by a direct parameter optimization method such as differential inclusion[8]), rendering the algorithm suitable for possible real-time applications.

With zero wind velocity, two locally optimal solutions exist, as shown in Figs. 2 and 3. The former is a bang–level–bang trajectory, and the elapsed time is 48.24 s; the latter is a bang–bang–bang type for which the elapsed time is 42.83 s and therefore is the global minimum. It turns out that, in this case, more switches do not yield any additional local minima.

Letting the wind velocity be 150 ft/s in the x direction only, i.e., $W_x = 150$ and $W_y = 0$, the new locally optimal solutions are shown in Figs. 4 and 5. Now, for the bang–bang–bang case, the elapsed time is 46.20 s, whereas in the bang–level–bang case, it takes only 46.19 s to reach the target.

V. Conclusions

The wind effect improves the performance in one case and degrades it in the other, rendering both trajectories comparable in terms of the transition time. We conclude that winds may change the character of the optimal trajectory and almost always affect the cost, either improving or degrading the performance.

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Identification of Linear Model Parameters and Uncertainties for an Aircraft Turbofan Engine

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Introduction

THE identification of turbofan engine dynamics as a multivariable piecewise linear model, along with modeling uncertainties from turbofan engine control system nonlinear model data, is necessary for developing advanced algorithms of sensor/actuator failure detection/isolation/accommodation and robust optimal control. Some gas turbine models have been identified. A number of papers and reports on engine identification and parameter estimation have been discussed in the survey by Merrill et al.[1] An algorithm based on least-squares estimation and nonlinear dynamic filtering was highlighted. The model was multivariable, and noise was introduced to simulate stochastic I/O data. The maximum likelihood method was applied to simulated open-loop and actual closed-loop engine data.[2] A nonlinear programming technique was used to estimate matrix parameters of a state-space aircraft model.[3] A two-step estimation approach for nonlinear systems with unknown process and measurement-noise covariances was applied to simulated aircraft response data.[4] A parameter identification algorithm based on smoothing test data with successively improved sets of system model parameters[5] and the maximum likelihood method using the V-Lambda square-root filtering technique[6] decrease the numerical difficulties. There also were some identification efforts in the frequency domain[7] and even in the quantification of parametric uncertainty.[8] In the time domain, the identification of model parameters and associated uncertainties was made for robust[9] or reconfigurable[10] control design.

In this work, a new estimation procedure is used to estimate both unknown piecewise linear model matrix parameters and matrix parameters limiting modeling uncertainties. These uncertainties are the differences between nonlinear and linear models and not uncertainties in the nonlinear model itself. Therefore, the purpose of this work was the development of an identification method for linear model parameters and uncertainties in the time domain from nonlinear simulation data, its application, and demonstration using a real case.

Identification of a Piecewise Linear Model

The detailed nonlinear models of different aircraft engines and, in particular, of a modern twin-spool afterburning turbofan engine, may be presented approximately as

$$\dot{\boldsymbol{x}}^{\text{abs}} = \boldsymbol{f}(\boldsymbol{x}^{\text{abs}}, \boldsymbol{u}^{\text{abs}}, \text{ALT}, \text{MN}) \tag{1}$$

$$\boldsymbol{y}^{\text{abs}} = \boldsymbol{g}(\boldsymbol{x}^{\text{abs}}, \boldsymbol{u}^{\text{abs}}, \text{ALT}, \text{MN}) \tag{2}$$

where $\boldsymbol{x}^{\text{abs}}$ is a state vector, $\boldsymbol{u}^{\text{abs}}$ is a control vector, and $\boldsymbol{y}^{\text{abs}}$ is an output vector; and system functions $\boldsymbol{f}$ and $\boldsymbol{g}$ are nonlinear real-value vector functions. A model of this type is used to obtain response data at arbitrary operating points. Three variables—altitude (ALT), Mach number (MN), and power-lever angle (PLA)—are sufficient to completely define each operating point.

A set of linear models, along with a description of modeling errors (uncertainties) at several selected operating points, is a piecewise linear model that can describe engine behavior at all operating points. Each linear model is assumed to be described by the following discretized equations:

$$\boldsymbol{x}(k+1) = (A \pm \Delta A)\boldsymbol{x}(k) + B\boldsymbol{u}(k) \tag{3}$$

$$\boldsymbol{y}(k) = (C \pm \Delta C)\boldsymbol{x}(k) + D\boldsymbol{u}(k) \tag{4}$$

where $\boldsymbol{x}$ is an n-dimensional state deviation vector, $\boldsymbol{u}$ is an m-dimensional control deviation vector, and $\boldsymbol{y}$ is an l-dimensional output deviation vector that does not include state deviation vector components. In other words, $\boldsymbol{x} = \boldsymbol{x}^{\text{abs}} - \boldsymbol{x}^{\text{sta}}$, $\boldsymbol{u} = \boldsymbol{u}^{\text{abs}} - \boldsymbol{u}^{\text{sta}}$, and $\boldsymbol{y} = \boldsymbol{y}^{\text{abs}} - \boldsymbol{y}^{\text{sta}}$ are the vectors of deviations from steady-state values; $\boldsymbol{x}^{\text{sta}}$, $\boldsymbol{u}^{\text{sta}}$, and $\boldsymbol{y}^{\text{sta}}$ are the vectors of steady-state values that correspond to one of the operating points; $k = 0, \ldots, N-1$ corresponds to $t_0, \ldots, t_{N-1}$, at $t_{k+1} = t_k + \Delta t$; and Δt is a uniform sampling time.

We want to estimate unknown matrices A, B, C, and D and unknown matrices with positive elements $\Delta A^{\max}$, $\Delta C^{\max}$ based on the assumption that

$$0 \le \Delta a_{ij} \le \Delta a_{ij}^{\max}, \qquad i = 1, \ldots, n, \quad j = 1, \ldots, n \tag{5}$$

$$0 \le \Delta c_{ij} \le \Delta c_{ij}^{\max}, \qquad i = 1, \ldots, l, \quad j = 1, \ldots, n \tag{6}$$

We first determine static model matrix parameters for each selected point of the static curve. If the states are steady near the point,

$$\boldsymbol{x} = K^x \boldsymbol{u} \tag{7}$$

$$\boldsymbol{y} = K^y \boldsymbol{u} \tag{8}$$

and the columns of matrices K^x and K^y may be determined from the detailed nonlinear engine simulation of response to given scalar step inputs by each component of the control vector. Let us assume that

$$B = (I - A)K^x \tag{9}$$

$$D = K^y - CK^x \tag{10}$$

The matrices A, C and $\Delta A^{\max}, \Delta C^{\max}$ then may be estimated from the discrete data of the nonlinear model simulated response $\tilde{\boldsymbol{x}}(k), k = 0, \ldots, N$ and $\tilde{\boldsymbol{y}}(k), k = 0, \ldots, N-1$ to $\tilde{\boldsymbol{u}}(k), k = 0, \ldots, N-1$ corresponding to PLA step inputs near a selected operating point. Obviously, we cannot use the data corresponding to $k = 0$ if $\tilde{\boldsymbol{x}}(k) = 0$. This estimation implies solving a linear programming problem for each row of matrices A, $\Delta A^{\max}$ (A_i, $\Delta A_i^{\max}$, $i = 1, \ldots, n$):

$$A_i, \Delta A_i^{\max} : \min \left\{ \delta, \delta \ge \sum_{j=1}^{n} \Delta a_{ij}^{\max} |\tilde{x}_j(k)|, \quad k = 1, \ldots, N-1 \right.$$

$$\sum_{j=1}^{n} \Delta a_{ij}^{\max} |\tilde{x}_j(k)| \ge \left| \tilde{x}_i(k+1) - A_i \tilde{\boldsymbol{x}}(k) - \left(K_i^x - A_i K^x\right) \tilde{\boldsymbol{u}}(k) \right|, \quad k = 1, \ldots, N-1$$

$$\left. \Delta a_{ij}^{\max} \ge 0, \quad j = 1, \ldots, n \right\} \tag{11}$$

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